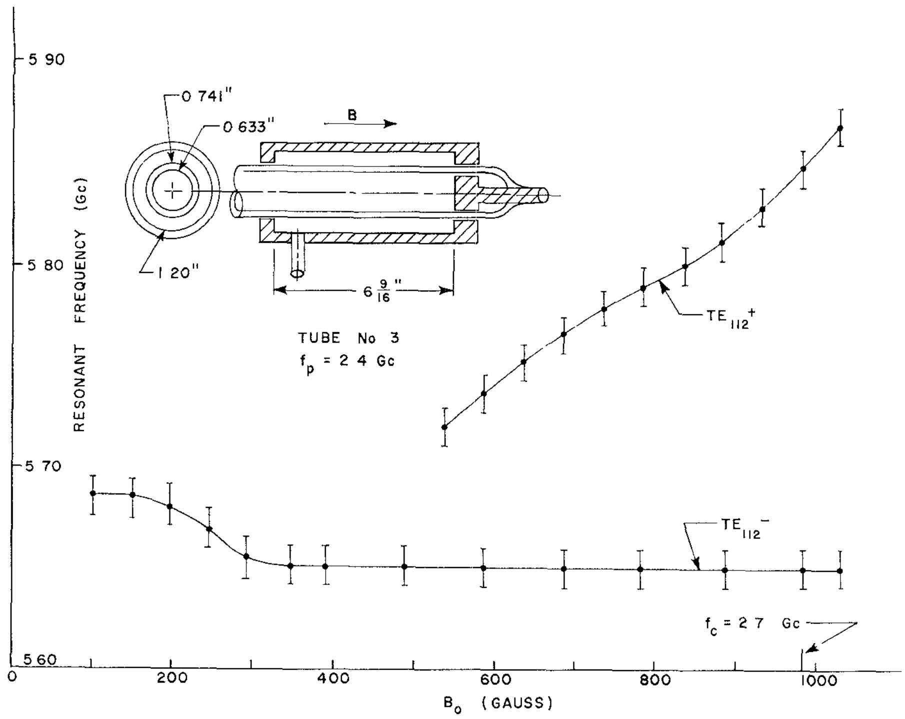

Fig. 3. Variation of resonant frequency of TE_{112}^+ and TE_{112}^- resonances with magnetic field. $f_p = 2.4$ Gc/s.

the "shorting-plane" at the cathode-end of the system is merely a metal ring filling the gap between the glass tube and the copper waveguide, so that the effective position of the reflecting plane at this end may vary slightly with frequency.

The plasma frequency f_p of 2.4 Gc/s indicated in Fig. 1 was determined by the well-known cavity perturbation technique [6]. This measurement required an auxiliary cavity resonant in the TM_{010} mode into which the discharge tube could be inserted coaxially [5].

Figure 3 shows the dependence of the resonant frequencies of the TE_{112}^+ and TE_{112}^- modes on the magnetic field with the resonant system at its maximum length. Careful checking shows that any TE_{11m}^+ resonance will have the same type of dependence on magnetic field as the TE_{112}^+ resonance shown in Fig. 3. Any TE_{11m}^- resonance will have the same type of dependence as the TE_{112}^- resonance.

It is seen that the TE_{112}^- resonance is insensitive to magnetic field except at low fields, while the TE_{112}^+ resonance is quite sensitive to magnetic field. Below 500 gauss it was not possible to detect the TE_{112}^+ resonance because it was masked by the TE_{112}^-, a stronger resonance. Above 1050 gauss it was again not possible to measure the TE_{112}^+ because here it was masked by the TE_{113}^- resonance. These results are in qualitative agreement with the work of Bevc and Everhart [4].

In addition to the signal power appearing at the output of the detection probe, there was also a comparable amount of noise power. The noise power was always nearly proportional to signal power. The presence of noise power did not lessen the accuracy of the measurements because the noise power was a minimum when the signal power was a minimum.

It has been the purpose of this correspondence to demonstrate that the propagation constants of the quasi-TE_{11}^+ and quasi-TE_{11}^- modes may be found with good accuracy by measuring positions of field nodes of resonances derived from these modes. We have also shown of course, that the TE_{11m}^+ and TE_{11m}^- resonances do exist and can be excited easily, at least when the plasma is the positive column of a mercury vapor discharge. While considerable noise power is present, it will not seriously affect the accuracy of the measurements.

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Design of Comb-Line Band-Pass Filters

Comb-line band-pass filters are filters using direct-coupled quarter-wave TEM resonators with adjacent resonators having the *same* open- and short-circuit reference planes. This is in contrast to interdigital band-pass filters where adjacent resonators *alternate* the open- and short-circuit reference planes. Comb-line band-pass filters can be realized using either strip (rectangular center conductors) or *slab* (round center conductors) transmission line resonators. When partitions are employed between adjacent resonators, the comb-line filter structure evolves into a coaxial filter structure. Consequently, comb-line and coaxial nomenclature will be used interchangeably herein. In this correspondence, the published design procedure for comb-line band-pass filters will be related to existing narrow-band filter theory. Certain aspects of the evolution of comb-line structures into coaxial structures will also be discussed.

A procedure has been presented applicable to the design of comb-line band-pass filters of narrow or moderate bandwidth [1]. In this filter structure, use of coupled quarter-wave lines of uniform cross section has been shown to result in an all-stop network. Adjacent resonators are decoupled due to the cancellation of equal magnetic and electric fields in phase opposition. To achieve band-pass filter behavior, resonator lines are foreshortened to electrical lengths less than ninety degrees, and resonance is obtained using lumped-capacitances at the resonator open-ends. Appreciable coupling between adjacent resonators is realized because the magnetic and electric couplings are no longer equal. In this case, the net coupling will be magnetic.

For narrow-band filters (i.e., bandwidth less than ten percent), it is convenient to describe the interstage couplings by coefficients of coupling [2] which can be readily measured by simple experimental procedures [3]. Assuming all interstage couplings to be equal, Matthaei's design equations [1] can be rewritten as follows, when $Y_{AK} = Y_A$:

$$\frac{C_{ij}}{\epsilon} = \frac{377 Y_A}{\sqrt{\epsilon_r}} \left(\frac{J_{ij}}{Y_A}\right) \tan\theta_0 \qquad (1)$$

$$\frac{J_{ij}}{Y_A} = \frac{w}{\omega_1'} \left(\frac{b}{Y_A}\right) \frac{1}{\sqrt{g_i g_j}} \qquad (2)$$

$$\frac{b}{Y_A} = \left(\frac{\cot\theta_0 + \theta_0 \csc^2\theta_0}{2}\right). \qquad (3)$$

Letting $w = 1/Q_T$, $\omega_1' = 1.0$, and

$$K_{ij} = \frac{1}{Q_T}\sqrt{\frac{1}{g_i g_j}},$$

and combining (1), (2), and (3), it can be shown that

$$\frac{C_{ij}}{\epsilon} = \frac{377 Y_A}{\sqrt{\epsilon_r}} K_{ij} f(\theta_0) \qquad (4)$$

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where

$$f(\theta_0) = \frac{\tan\theta_0(\cot\theta_0 + \theta_0 \csc^2\theta_0)}{2} = \frac{1}{2}\left[1 + \frac{2\theta_0}{\sin 2\theta_0}\right] \quad (5)$$

$$Q_T = f_0/\Delta f_{3dB} = \frac{\text{filter center frequency}}{\text{filter 3-dB bandwidth}}$$

= filter total Q, and

K_{ij} = absolute coefficient of coupling between ith and jth resonators.

If $C = C_i = C_j$, from Matthaei's [1] equations,

$$\frac{C}{\epsilon} = \frac{377 Y_A}{\sqrt{\epsilon_r}}\left[1 - 2\left(\frac{J_{ij}}{Y_A}\right)\tan\theta_0\right]. \quad (6)$$

Combining (2), (3), (4), and (6), it can be shown that

$$\frac{C}{\epsilon} = \frac{377 Y_A}{\sqrt{\epsilon_r}}\left[1 - K_{ij} f(\theta_0)\right] \quad (7)$$

where $f(\theta_0)$ is defined per (5). Dividing (4) by (7) and rearranging terms,

$$K_{ij} = \frac{1}{f(\theta_0)}\left[\frac{1}{\frac{C/\epsilon}{C_{ij}/\epsilon} + 2}\right]. \quad (8)$$

Using slab-line construction [4] (i.e., round center conductors between ground planes),

$$\frac{C}{\epsilon} = \frac{C_g}{\epsilon} \text{ and } \frac{C_{ij}}{\epsilon} = \frac{C_m}{\epsilon}.$$

Then

$$K_{ij} = \frac{1}{f(\theta_0)}\left[\frac{1}{\frac{C_g/\epsilon}{C_m/\epsilon} + 2}\right]. \quad (9)$$

$1/f(\theta_0)$ is tabulated below for common values of θ_0:

Degrees	$1/f(\theta_0)$
0	1.000
15	0.978
30	0.906
45	0.778
60	0.586
75	0.320
90	0.000

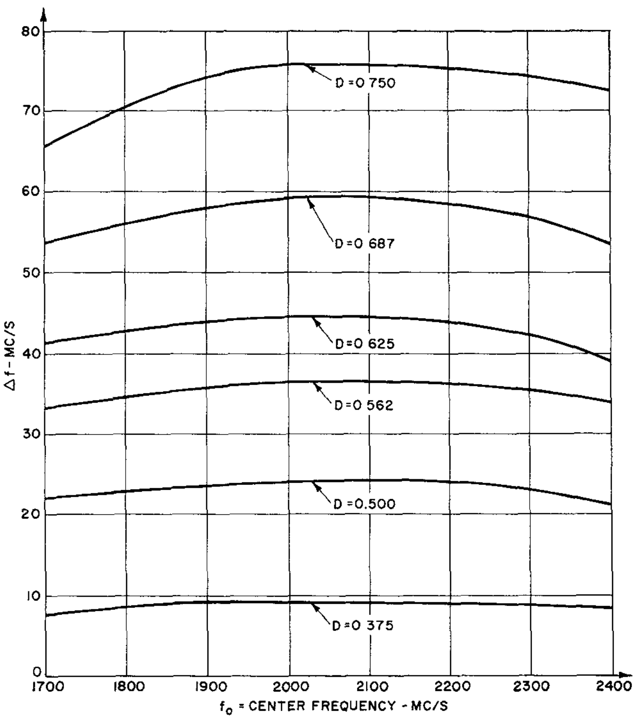

Fig. 1. Aperture coupling data. Coupling bandwidth vs. aperture diameter. Circular apertures 0.650" from plane of short coaxial resonators: 1.375"×0.750"/0.375 diameter.

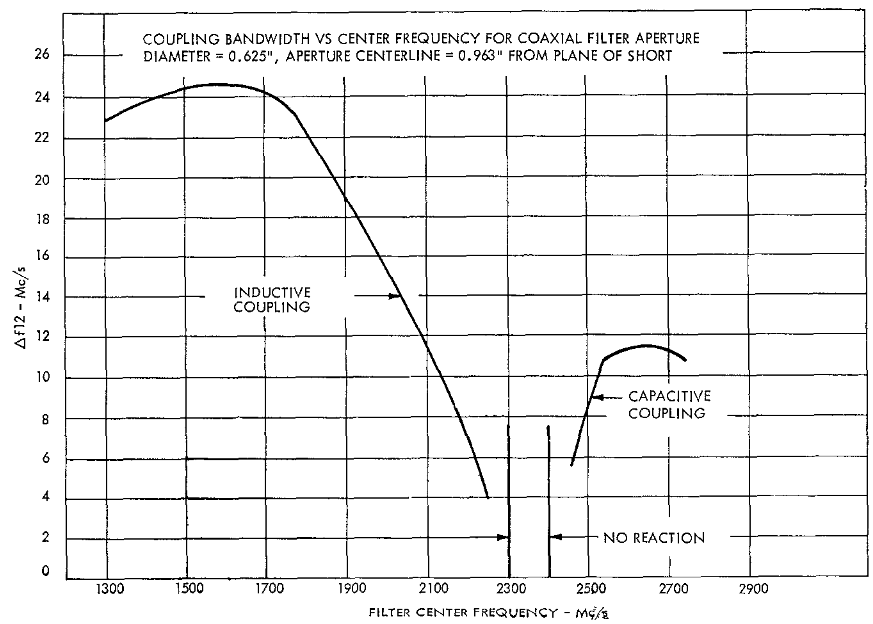

Fig. 2. Frequency sensitivity of coupling aperture.

Use of the recommended comb-line structure with foreshortened resonators results in a filter package that is smaller in one dimension; however, this foreshortening of the resonator line lengths degrades the resonator unloaded Q's and increases pass band insertion losses due to dissipation. Matthaei [1] has mentioned that foreshortened comb-line resonators have lower unloaded Q's than quarter-wave interdigital resonators of the same cross section. Experimental data for comb-line band-pass filters for $30° < \theta_0 < 90°$ has shown the following empirical relationship to be applicable for comb-line reson tors of constant cross section:

$$Q_{UL}(\text{effective}) \cong \sin^2(\theta_0) Q_{UL}, \quad (10)$$

where Q_{UL} (effective) is the unloaded Q of the foreshortened resonator, and Q_{UL} is the unloaded Q of the quarter-wave resonator. When $\theta_0 = 45$ degrees, the effective unloaded Q is reduced by a factor of 2. To use comb-line filters with full quarter-wave resonators, different coupling techniques can be employed.

Interstage coupling in comb-line filters can be realized using partitions between adjacent resonators with coupling apertures in the partitions [5], [6]. Circular coupling apertures are usually located for minimum frequency sensitivity [7], while aperture diameters are determined experimentally to

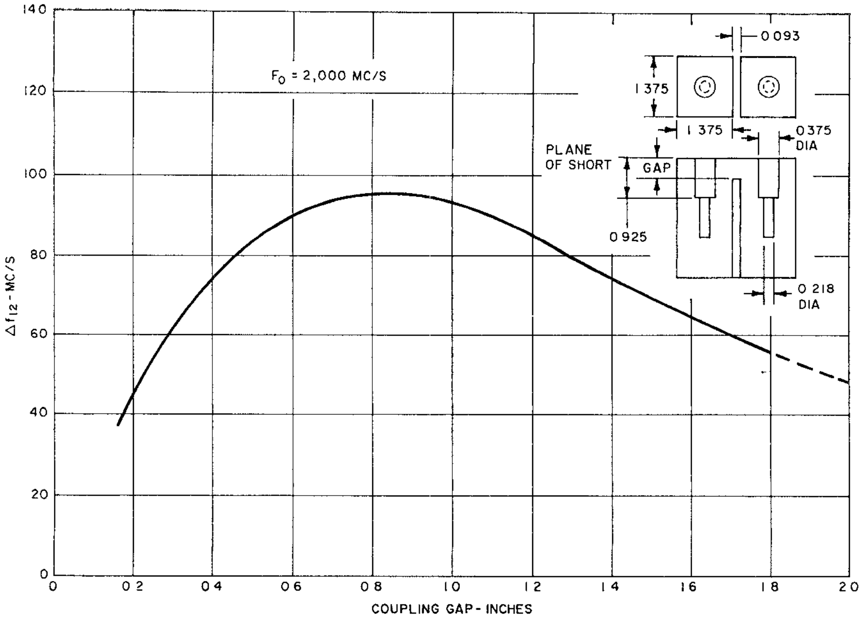

Fig. 3. Coaxial filter coupling data (square cross section outer conductors).

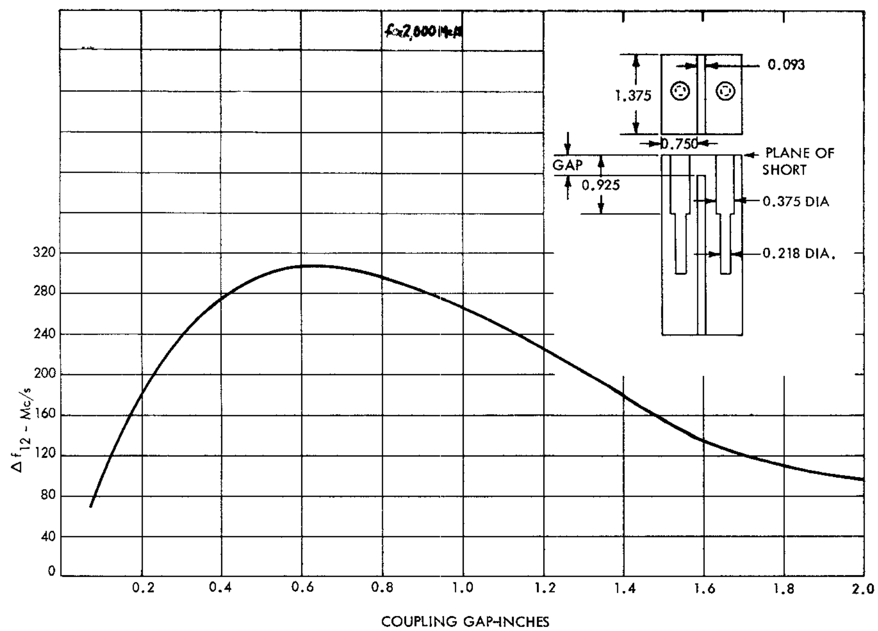

Fig. 4. Coaxial filter coupling data (rectangular cross section outer conductors).

obtain the desired coefficients of coupling. Typical experimental curves of coupling bandwidth vs. center frequency for various aperture diameters are shown in Fig. 1 for a comb-line filter structure. For this aperture location, the net coupling will be magnetic (i.e., inductive). It should be noted that absolute coefficient of coupling $K_{12} \cong \Delta f_{12}/f_0$, where Δf_{12} is the coupling bandwidth, vs. center frequency is shown in Fig. 2 for a somewhat different aperture location. When the aperture is a certain electrical distance from the plane of the short circuit, a cancellation of magnetic and electric fields will occur, and adjacent resonators are effectively decoupled. This is the region of no reaction of Fig. 2. For comb-line filters using partitions between adjacent resonators, some degradation in resonator unloaded Q's will occur due to resistive losses in the partition walls.

Wall losses can be substantially avoided using partial partitions that shield (decouple) adjacent resonators only in the regions of strong electric fields. Figures 3 and 4 show coupling bandwidth Δf_{12} as a function of coupling gap for comb-line filter structures of two different resonator cross sections. With resonators using square outer conductor cross section (see Fig. 3), a maximum coupling bandwidth of 4.75 percent has been obtained. With resonators using rectangular outer conductor cross section, (see Fig. 4), a maximum coupling bandwidth of 14.7 percent has been obtained. In both cases, as the coupling gap increases, Δf_{12} increases until the incremental increase in electric coupling overtakes the incremental increase in magnetic coupling. With no partitions, the net coupling is still magnetic, and complete cancellation of magnetic and electric couplings does not occur. This is due to the use of compound center conductors making resonator cross sections nonuniform, and magnetic and electric couplings unequal.

Under some circumstances, use of partitions can have other advantages. A family of comb-line band-pass filters with different response shapes and/or bandwidths can be developed with overall filter size depending only upon center frequency and number of resonators. Direct-coupled comb-line filters will require variations in resonator foreshortening and/or adjacent resonator separation to implement changes in filter response shapes and/or bandwidths. The use of partitions can sometimes help in suppressing some of the filter spurious responses.

The comments of Dishal [8] with respect to narrow-band interdigital filters are also applicable to narrow-band comb-line filters. Use of auxiliary rods for filter input /output couplings are unnecessary; and coupling mechanisms such as loops, probes, or direct taps should be employed in obtaining the required singly loaded Q's for the input/output resonators [5].

For filters of moderate percentage bandwidth (i.e., ten percent to twenty percent), use of Matthaei's design equations is more appropriate than utilizing the narrow-band concepts of coefficients of coupling and singly loaded Q. In this case, reductions in unloaded Q's, due to the foreshortening of resonator lengths, will be less significant.

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